THE EFFECT OF A SELF-INDUCED ELECTRIC FIELD ON ELECTRON BEAM DIFFERENTIAL SPECTRA IN FLARING ATMOSPHERES

V. V. Zharkova and M. Gordovskyy

The School of Informatics, University of Bradford, Bradford, BD7 1DP

ABSTRACT

The effect of a self-induced electric field on beam electron distributions is investigated analytically. The electric field induced by a precipitating beam is variable in depth being a constant at upper atmospheric layers and falling from some 'turning point' that occurs either in the corona for intense soft beams or in the chromosphere for weak hard beams. The magnitude of a constant electric field is about $10^{-7}$ V/cm for a weak beam the initial flux of $10^8$ erg/cm²/s with spectral index $\gamma = 3$ and reaches $10^{-3}$ V/cm for a beam with the flux of $10^{12}$ and index 7. The analytical solutions for a constant and decreasing with depth electric fields are presented, the characteristic 'electric' stopping depths are calculated and compared with the 'collisional' ones. The constant self-induced electric field is found to decelerate precipitating electrons significantly reducing their number at upper precipitation depths in the corona that results in the differential spectra flattening at lower energies (< 100 KeV). The electric field decrease with depth reduces the electron deceleration that allows beam electrons to precipitate into deeper atmospheric layers in comparison with a constant one.

Key words: Sun: solar flares - particle beams, Plasma - collisional losses; Plasma - Ohmic losses, Solar flares - hard X-ray emission.

1. INTRODUCTION

The effect of the induced electric field on the dynamics of non-thermal electron beams was investigated by (3) by considering the energy and momentum conservation equations and including collisional and Ohmic heatings caused by the electric field induced by a precipitating electron beam and ignoring return current instabilities leading to a generation of plasma turbulence (2). The precipitating beam was revealed to have a noticeable effect of the induced electric field that significantly increases the energy deposited by such a beam in the chromosphere in comparison with those in pure collisions leading to significantly increased hard X-ray emission or even its saturation for the beams with high initial fluxes.

This approach was considered by using the time-dependent Fokker-Plank equation for beam electrons precipitating into a flaring atmosphere with exponential increase of density and decrease of temperature and taking into account these two energy losses mechanisms: collisions with the ambient particles and Ohmic losses of beam electrons in self-induced electric field (4; 6). The higher is the beam energy flux and the higher its spectral index the bigger is the difference between the spectral indices of lower and higher energy parts in HXR photon spectra (7). However, a precise mechanism of the electric field effect on electron beam kinetics and resulting hard X-ray emission and on the electron mean flux still remains unclear.

The motivation for the current research is to obtain simple kinetic solutions for electron beam differential spectra in a pure electric field approach for various electric field profiles. The analytical solutions of the continuity equation are described in Section 2 and the conclusions are drawn in Section 3.

2. RESULTS AND DISCUSSION

2.1. Depth distributions of a self-induced electric field

The kinetics of very intense electron beams can be strongly affected by the self-induced electric field (6; 7). The electric field distributions with depth found from full kinetic solutions are plotted in Figure 1 for electron beams with $\gamma = 3$ and the initial flux of $10^8$ erg/cm²/s (upper plot), $10^{10}$ (middle plot) and $10^{12}$ (lower plot) (Zharkova and Gordovskyy, 2005).

Generally speaking, in flaring atmospheres the induced electric field is not constant at the whole precipitation depth as it is shown in Figures 1 taken from the full kinetic simulations of equation (2) by (7) measured in the
units of a local Dreicer field \( \varepsilon_D = \frac{\text{2n}_e^2 n_e}{\text{ks} \times T} \) where T and \( n \) are local temperature and density, \( n_e \Lambda \) is the Coulomb logarithm and e is the electron charge.

The self-induced electric field has a nearly constant part at upper coronal levels and strongly falls with depth at the lower corona and transition level (see Figure 1). For weaker harder beams the induced electric field is nearly constant at the coronal and upper chromospheric levels downwards to a column depth of a few units of 10\(^{20}\) cm\(^{-2}\) and then smoothly decreases with depth towards the photosphere (Figure 1a). For more intense beams the magnitude of a constant electric field increases proportionally to the initial flux increase but it remains constant only in the coronal levels of about 10\(^{19}\) cm\(^{-2}\) (for the initial flux of 10\(^{10}\)) (Figure 1b) or even about 10\(^{18}\) cm\(^{-2}\) (for 10\(^{12}\) erg/cm\(^2\)/s) (Figure 1c) after which it sharply decreases. The level of decrease with depth depends on beam spectral indices being steeper for softer beams (with higher indices of 7 or 5) than for harder ones (lower index of 3) as it is shown by solid and dashed lines in Figure 1a-c.

### 2.2. Kinetic solutions for a pure electric field

Let us assume that an electric field (constant or variable) is present during precipitation of a power law electron beam and this field is not affected by the precipitating ions. For the sake of simplicity, let us assume that the beam is injected in the direction \( \mu = 1 \) with \( \delta \)-like pitch angular distribution and the particles can either precipitate downwards (\( \mu = 1 \)) or, after they fully lose their energy, move backwards to the source in the corona (\( \mu = -1 \)).

The main features of an electron beam kinetics in a flaring atmosphere can be considered analytically by solving the continuity equation:

\[
\frac{\partial}{\partial x} [V \cdot N(x, E)] + \frac{\partial}{\partial E} \left[ \left( \frac{dE}{dx} \right) V \cdot N(x, E) \right] = 0, \tag{1}
\]

for energy losses \( \frac{dE}{dx} \) in a pure electric field defined as (Emslie, 1980):

\[
\frac{dE}{dx} = -e\varepsilon, \tag{2}
\]

where \( x \) is a linear precipitation depth.

Let us impose that the initial beam spectrum is a power law one with a spectral index \( \gamma \) (for the flux) giving the initial electron density as:

\[
N(E, 0) = K E^{-\gamma-0.5} \Theta(E - E_{\text{low}}) \tag{3}
\]

and find the solutions for a constant and variable electric field.

#### 2.2.1. Constant electric field

For a constant electric field (\( \varepsilon = \text{const} \)), beam differential densities can be found from the continuity equation (5) using formula (6) as follows:

\[
N(E, x) = K \varepsilon^{-0.5} \cdot (E + e\varepsilon x)^{-\gamma} \times \Theta(E - E_{\text{low}} + e\varepsilon x) \Theta(E_{\text{upp}} - e\varepsilon x - E), \tag{4}
\]

for \( \mu = +1 \)

\[
N(E, x) = K \cdot \varepsilon^{-0.5} (e\varepsilon x)^{-\gamma} \times \Theta(E - E_{\text{low}} - e\varepsilon x) \Theta(E_{\text{upp}} + e\varepsilon x - E), \tag{5}
\]

for \( \mu = -1 \)

The differential spectra are asymmetric for precipitating particles that is illustrated by Fig. 2 for electron beams with the initial flux of 10\(^{10}\) and 10\(^{12}\) and the initial index \( \gamma = 3 \) (for \( \mu = 1 \) (a) and \( \mu = -1 \) (b), respectively) (Zharkova and Gordovskyy, 2006). The precipitating electrons, after losing their energy completely in the electric field, become accelerated by this field in the opposite direction and return back to the source in the corona. The energy they gain during the acceleration depends on their original energy, or their stopping depth, from which they start their acceleration, and it is smaller for lower energy electrons and higher for higher energy ones.

For a precipitating beam the differential densities depend on a linear precipitation depth \( x \) that is associated with a column depth and the ambient plasma density as \( x \approx \frac{E}{e\varepsilon} \). This means that, unlike the electron densities for pure collisional losses being dependent only on a column depth, the number of beam electrons losing its energy in electric field is dependent on a depth distribution of the ambient plasma particles. As expected, for a constant electric field in a flaring atmosphere with the exponentially increasing density the beam electrons steadily lose the same amount \( e\varepsilon x \) of their energy with a precipitating distance \( x \), and with every step of their precipitation for lower energy electrons their loss becomes more noticeable comparing to their energy. Therefore, at deeper atmospheric layers the electron differential spectra steadily decrease at lower energies until all electrons are fully decelerated by the electric field. Obviously, the lower energy electrons lose their energy at the atmospheric levels higher than more energetic ones. Let us introduce "an electric" stopping depth \( x_{e} \), at which the electron with energy \( E \) loses it completely, i.e.

\[
x_{e} = \frac{E}{e\varepsilon}, \quad \text{or} \quad \varepsilon_{e} = \frac{E}{e\varepsilon} \frac{1}{n} \tag{6}
\]
For example, in the exponential atmosphere used in this study for the electric field of $10^{-04}$ $\text{erg/cm}^2\text{s}^{-1}$ electrons with the initial energies of 30 and 300 keV will lose their energy completely at the ‘electric’linear stopping depths, or at the associated ‘electric’ column depths of $5 \times 10^{17}$ and $6 \times 10^{25}$ cm$^{-2}$, respectively. Hence, depending on a strength of the induced electric field $\mathcal{E}$, this ‘electric’ stopping depth can be much shorter than the collisional one leading to lower energy beam electrons returning to the injection source in the corona well before they reach the depth where they fully lose their energy in collisions. In Table 1 (Zharkova and Gridivsky, 2006) we presented a comparison of the ‘collisional’ and ‘electric’ stopping depths calculated for the electric field magnitudes relevant for the accepted beam parameters and the exponential atmospheres. Intense beams inducing a higher electric field are dominated by the electric field losses and their stopping depths are located very high in the corona (about or lower $10^{18}$ $\text{cm}^{-2}$) close to the injection source at $(10^{17}$ $\text{cm}^{-2})$.

### 2.2.2. Variable and combined electric field

The full kinetic solutions (7) has shown that the induced electric field is not constant throughout the whole flaring atmosphere but only at higher levels to the depth after which it falls exponentially (linearly, parabolically or steeper) (see Fig.1 above). The ‘turning’ depth where electric field starts decreasing, is very important for understanding the resulting electron and photon spectra.

Let us obtain pure electric solutions for a few cases of Ohmic energy losses in a variable electric field assuming it decreases with depth as $\mathcal{E} = \mathcal{E}_0 \left(\frac{x}{x_t} + 1\right)^{-k}$ where $\mathcal{E}_0$ is a constant electric field, $x_t$ is a turning depth and $x \geq x_t$.

for $k \geq 1$ they can be defined as follows:

\[
N(E, x) = \frac{K}{\sqrt{\mathcal{E}}} \left(E + \frac{\mathcal{E}_0 x_t}{k-1} \left[1 - \left(\frac{x}{x_t}\right)^{1-k}\right]^{-\gamma}\right) \times \\
\times \Theta \left(E - E_{\text{low}} + \frac{\mathcal{E}_0 x_t}{k-1} \left[1 - \left(\frac{x}{x_t}\right)^{1-k}\right]\right) \times \\
\times \Theta \left(E_{\text{upp}} - \frac{\mathcal{E}_0 x_t}{k-1} \left[1 - \left(\frac{x}{x_t}\right)^{1-k}\right] - E\right) (7)
\]

The precipitating electrons with lower energies, whose ‘electric’ stopping depth is above the depth of a turning point, will lose less their energy in deceleration by a decreasing electric field that results in less flattening of the differential energy spectra as demonstrated in Figure 2 for a beam with the initial flux of $10^{12}$ $\text{erg/cm}^2\text{s}^{-1}$ and $\gamma = 3$ and the electric field constant (a) and decreasing as $\sim x^{-1}$ (b) and $\sim x^{-2}$ (c) from the turning point occurring at the first depth step.
Figure 2. The differential spectra of beam electrons with $\gamma = 3$ calculated in a pure electric approach for a combined electric field with a constant part of $5 \times 10^{-5}$ V/cm, the turning point at $i=4$ and with a variable field dropping as $1/x^2$ (a), $1/x^3$ (b) and $1/x^5$ (c).

Figure 3. The differential spectra of beam electrons with $\gamma = 3$ calculated in a pure electric approach for a combined electric field falling as $1/x^2$ with turning point at $i=4$ and with a constant part of $5 \times 10^{-5}$ V/cm (a) and $4 \times 10^{-4}$ V/cm (b).
In Figure 3 the differential spectra are plotted for a combined electric field with the constant electric field increased from $5 \times 10^{-3}$ V/cm (a) to $4 \times 10^{-1}$ V/cm (b) and the variable part decreasing as $\sim x^{-2}$. This shows a faster decrease in electron numbers for higher electric field that leads to a bigger flattening of their differential spectra if the constant electric field is higher.

3. THE CONCLUSIONS

In the present paper we present the analytical solutions of the continuity equation for pure Ohmic energy losses in the induced electric field for the beams with initial energy fluxes of $10^8$ and $10^{12}$ erg/cm$^2$/s and the initial spectral indices of 3, 5 and 7.

The differential densities found from a pure electric field approach for electron beams with the initial indices $\gamma = 3$ (for $\mu = 1$ (a) and $\mu = -1$ (b), respectively) are asymmetric for precipitating and returning particles. The electric stopping depth for the electrons with energy $E$ is $x_E = \frac{E}{eE}$.

For a constant electric field in a flaring atmosphere with the exponentially increasing density the beam electrons steadily lose the same amount $eE x$ of their energy with a precipitating distance $x$.

A deceleration by an electric field is found to affect lower energy electrons ($< 100$ KeV) by significantly reducing their number at the upper precipitation depths in the corona resulting in the spectra flattening towards lower energies. For the electric field decreasing with depth (linearly or parabolically) the differential spectra become less flat, or their index decreases less for beams with the same parameters.

REFERENCES